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An experimental quadrature modulation system for video signals

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AN EXPERIMENTAL QUADRATURE MODULATION SYSTEM FOR VIDEO SIGNALS

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Research Department Report No. 1971/30

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AN EXPERIMENTAL QUADRATURE MODULATION SYSTEM FOR VIDEO SIGNALS

Summary

An experimental two-channel quadrature modulation system operating at v.h.f. has been developed. This is used to illustrate the performance that may be expected of a practical system for transmission of wide-band signals in the u.h.f. or s.h.f. bands.

This method of modulation enables two signals to be transmitted at the same carrier frequency. Two carriers of identical frequency, but displaced in phase by 90°, are modulated separately and then added to form the quadrature-modulated signal. At the receiving terminal the two modulating signals are separated in synchronous detectors which are fed with reference carrier signals in phase quadrature. In order that the modulating signals can be separated completely, the reference carrier signals must be coherent with the quadrature-modulated signals. In the experimental system a pilot modulation signal (6 MHz) is transmitted to ensure the correct generation of the reference carrier signals.

The theory and design of the system is considered, and measurements using independent video signals applied to the two modulators showed that crosstalk and distortion could be kept at a low level. The phase-locked oscillator used in the demodulator was capable of generating the reference carrier signals with an accuracy of 0·5 degrees over a range of input signal frequencies of 200 kHz.

1. Introduction

A great deal of work has recently been carried out on the use of pulse code modulation methods for transmission of sound and television signals. At present this technique is likely to be too expensive for domestic use, but the rapid developments in integrated circuits may make the system viable. Pulse coding offers advantages over conventional analogue methods for national and international distribution of programmes. If such a signal is to be transmitted most efficiently over an r.f. link, the modulation system must be chosen to provide the required signal-to-noise ratio in the minimum r.f. bandwidth.

Quadrature modulation enables two separate signals to be transmitted at the same carrier frequency. The two signals are modulated independently on two carriers which are in phase quadrature and the r.f. bandwidth required to transmit the two signals is the same as that required for transmitting one signal by double-sideband amplitude modulation. This method of modulation is used for example for transmitting the two colour-difference signals on a single sub-carrier in the NTSC and PAL colour television systems.

For the transmission of digital information the signal could be split into two multi-level pulse signals for modulating the quadrature components. Hybrid pulse coding[1,2] could also be transmitted using quadrature modulation.* This method of coding requires the simultaneous transmission of a quantised (possibly pulse-coded) signal, and an analogue signal proportional to the difference between the quantised signal and the original waveform. It is important when using hybrid pulse coding that no timing error exists between the quantised and analogue components of the signal at the receiving terminal. Quadrature modulation of the two signals at one carrier frequency ensures that the propagation time of the two channels is exactly equal. Experimental work on this aspect is not, however, covered by the present report.

In order to investigate the practical limitations of wide-band quadrature modulation, and the difficulties in its instrumentation, an experimental modulator and demodulator were constructed. This equipment was designed to modulate two 5·5 MHz video signals on a carrier frequency of 57 MHz. This frequency represents an intermediate frequency that could be used in a practical u.h.f. or microwave transmission system. A description of the experimental equipment is given in Section 3.

2. Principles of quadrature modulation

A quadrature-modulated signal may be regarded as two independent signals which are carried simultaneously by the same channel. Two carriers of identical frequency are generated in phase quadrature; they are independently amplitude modulated and added to produce the quadrature-modulated signal. In order to conserve transmitter power, balanced modulation may be employed.

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* This suggestion was first made by G.G. Gouriet.

The two modulating signals are separated in the demodulator by feeding the quadrature-modulated signal into two synchronous detectors which are supplied with separate reference carrier signals in phase quadrature. The process of demodulation is accomplished in each detector by multiplication of the quadrature-modulated signal by the reference carrier.

Considering the two modulator carriers to be cos $\omega_c t$ and sin $\omega_c t$ and the two modulating signals $f_1(t)$ and $f_2(t)$, the quadrature-modulated signal may be represented by

$$f_1(t)\cos\omega_c t + f_2(t)\sin\omega_c t$$

If the demodulator reference carriers are $\cos(\omega_c t + \phi)$ and $\sin(\omega_c t + \phi)$ the output of one detector will be (ignoring constant multipliers)

$$\cos(\omega_c t + \phi)\,[f_1(t)\cos\omega_c t + f_2(t)\sin\omega_c t]$$

$$= f_1(t)\cos\phi - f_2(t)\sin\phi + f_1(t)\cos(2\omega_c t + \phi) + f_2(t)\sin(2\omega_c t + \phi)$$

The last two terms represent a signal at twice the carrier frequency, and they are removed in low pass filters following the detector.

If $\phi = 0$, i.e., the reference carrier is in phase with one of the quadrature modulation carriers, the detector output (after filtering) is $f_1(t)$.

Similarly the filtered output of the other detector is

$$f_2(t)\cos\phi + f_1(t)\sin\phi$$

which, if $\phi = 0$, becomes $f_2(t)$.

It can be seen that if a phase error, ϕ, exists between the reference carriers and the carriers of the quadrature-modulated signal, the two modulating signals will not be completely separated in the demodulator.

Assuming equal amplitudes of $f_1(t)$ and $f_2(t)$, the relative level of the unwanted signal, the so-called crosstalk, is tan ϕ.

Crosstalk is also introduced in a quadrature-modulated system if the amplitude of phase response of the transmission path as a function of frequency is not symmetrical about the carrier frequency. To illustrate this effect, the two quadrature components of the signal may be considered independently. For simplicity, it will be assumed that one carrier is modulated by a sinusoidal signal and the modulation of the other carrier is zero.

If a sinusoidal signal cos $\omega_m t$ is used to modulate a carrier cos $\omega_c t$ using suppressed-carrier modulation the output frequency spectrum consists of two equal sidebands: $\tfrac{1}{2}\cos(\omega_c + \omega_m)t + \tfrac{1}{2}\cos(\omega_c - \omega_m)t$. If these sidebands are subjected to different attenuations, x and y, in the transmission path, the received signal will be

$$\frac{x}{2}\cos(\omega_c + \omega_m)t + \frac{y}{2}\cos(\omega_c - \omega_m)t$$

$$= \tfrac{1}{2}(x + y)(\cos\omega_m t . \cos\omega_c t) - \tfrac{1}{2}(x - y)(\sin\omega_m t . \sin\omega_c t)$$

The expression $\tfrac{1}{2}(x + y)(\cos\omega_m t . \cos\omega_c t)$ represents the original signal, cos $\omega_m t$ modulated on the carrier cos $\omega_c t$, and subjected to the mean attenuation of the two sidebands, while $\tfrac{1}{2}(x - y)(\sin\omega_m t . \sin\omega_c t)$ represents a signal modulated on a carrier in phase quadrature with the original carrier. In a quadrature-modulated system the latter signal would represent crosstalk. It should also be noticed that the crosstalk, $[(x - y)/(x + y)]$ sin $\omega_m t$, has been subjected to a phase shift of 90° relative to the transmitted modulation.

If the two sidebands $\tfrac{1}{2}\cos(\omega_c + \omega_m)t$ and $\tfrac{1}{2}\cos(\omega_c - \omega_m)t$ are subjected to phase shifts of ϕ_1 and ϕ_2 relative to a signal at the carrier frequency the received signal will be

$$\tfrac{1}{2}\cos[(\omega_c + \omega_m)t + \phi_1] + \tfrac{1}{2}\cos[(\omega_c - \omega_m)t + \phi_2]$$

$$= \tfrac{1}{2}\cos\omega_c t\,[\cos\omega_m t . (\cos\phi_1 + \cos\phi_2) + \sin\omega_m t . (\sin\phi_2 - \sin\phi_1)]$$

$$+ \tfrac{1}{2}\sin\omega_c t\,[\sin\omega_m t . (\cos\phi_2 - \cos\phi_1) - \cos\omega_m t . (\sin\phi_1 + \sin\phi_2)]$$

It the phase response of the transmission path is antisymmetrical about the carrier frequency, i.e. $\phi_1 = -\phi_2$, the received signal is

$$\cos\omega_c t . \cos(\omega_m t + \phi_1)$$

In this case no crosstalk is introduced but the modulating signal will suffer group-delay distortion unless ϕ_1 is proportional to ω_m.

If the departure of the phase response from antisymmetry is small the crosstalk introduced is predominantly in phase with the modulation, i.e. if the modulation transmitted in one channel is cos $\omega_m t$, the crosstalk detected in the other channel of the receiver will be $k_1 \cos\omega_m t + k_2 \sin\omega_m t$, where $k_1 \gg k_2$.

3. Experimental equipment

3.1. The modulator

A block diagram of the modulator is shown in Fig. 1. The two video signals are applied through 5·7 MHz low-pass filters to two integrated-circuit balanced modulators. The 57 MHz carrier inputs to the modulators are driven in phase quadrature by using a quarter-wavelength of cable in one path.

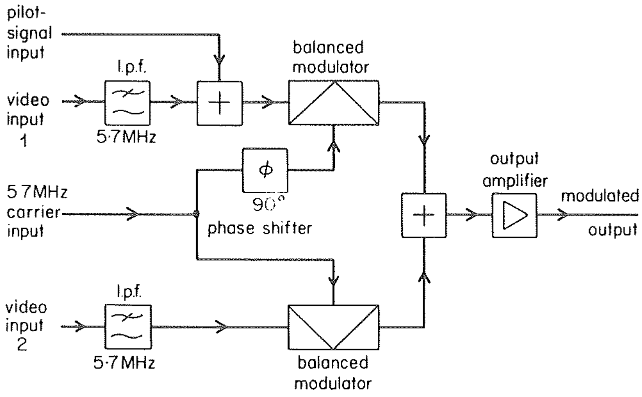


Fig. 1 - Block diagram of experimental modulator

The outputs of the two modulators are added and amplified to provide the quadrature-modulated output signal.

A 6 MHz pilot signal, added to the video input of one of the modulators, is required for the demodulator to lock an oscillator to the phase of the incoming signal carrier. This oscillator provides the reference carrier signals for the detectors. The low-pass filters in the video inputs to the modulators are used to reduce the level of any components of the video signal near 6 MHz, which could cause interference in the carrier locking system of the demodulator.

3.2. The demodulator

3.2.1. General

The block diagram of the demodulator is shown in Fig. 2. The amplified input signal is band-pass filtered and applied to two synchronous detectors, D1 and D2. The reference carrier signals are fed in phase quadrature to these detectors from a voltage-controlled oscillator, which is locked in phase with the input signal carrier by means of a d.c. control voltage derived from a phase-sensitive detector D3. The method is due to Costas.[3]

3.2.2. Video signal circuits

The video output signals from the detectors are amplified to a level of 1 volt peak-to-peak. 6 MHz notch filters are included in both amplifiers to reduce the r.m.s. level of the pilot signal at the video outputs to less than −40 dB with respect to peak-to-peak video.

3.2.3. Reference carrier locking circuits

Referring to Fig. 2, the 6 MHz pilot signal is separated from the video outputs of D1 and D2 by band-pass filters. The amplified pilot signals from both detectors are then applied to a phase-sensitive detector D3. If the phase of the reference carrier signal is correct there will be a 6 MHz pilot signal present only in the output of D1. A phase error in the reference carrier will cause a pilot signal to be present in the output of D2, this signal being in phase or in antiphase, depending on the direction of the phase error. The polarity of the output from D3 is dependent on the direction of this error. The d.c. and low-frequency components in the output from D3 are extracted by a filter and applied via an amplifier to the oscillator to provide phase control. A.C. feedback around the d.c. amplifier provides the required frequency characteristic to stabilise the phase-lock loop.

This type of phase-lock loop has two stable locked states, such that there is an equal probability that the oscillator will lock in the correct phase, or in antiphase. If the oscillator is locked in antiphase the signal outputs from the demodulators will both be inverted. An analysis of the design of the phase-lock loop is contained in the Appendix.

4. Performance of the experimental equipment

The system was tested with two independent 625-line Standard I television signals. The output of the modulator was connected directly to the input of the demodulator.

A crosstalk level between channels of better than −30 dB could be obtained without difficulty, and by careful adjustment of the demodulator it was found to be possible to reduce this to −40 dB. The remaining crosstalk was found to be dependent on the signal level, indicating non-linearity of the modulator or demodulator. The crosstalk was also to a lesser extent frequency-dependent. This effect was probably caused by slight asymmetry in the frequency response of the demodulator input stage.

The r.m.s. noise generated in the modulator and demodulator was −46 dB with respect to the peak-to-peak picture signal at the output of the demodulator.

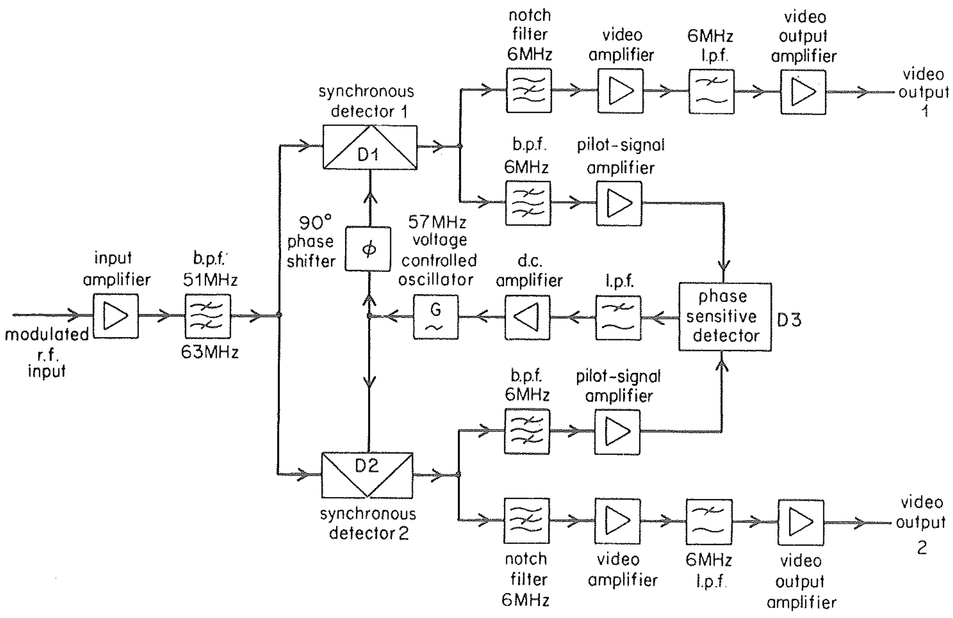
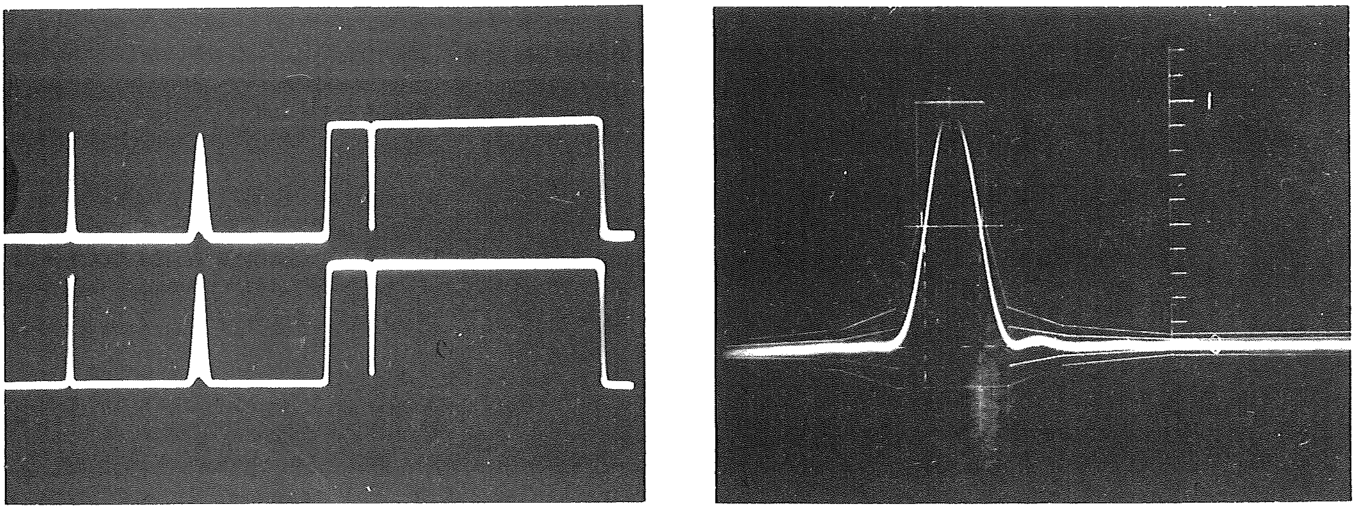


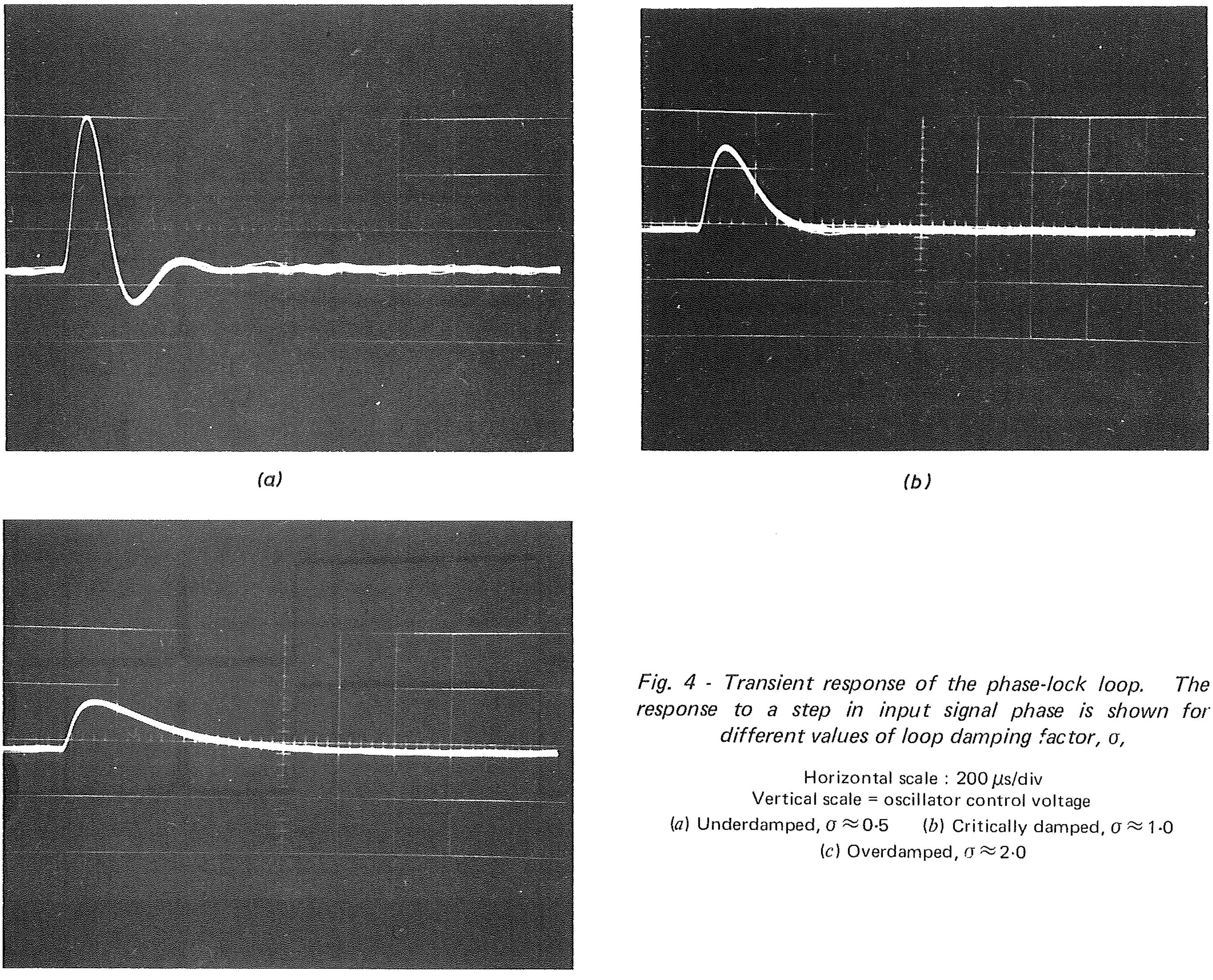
Fig. 2 - Block diagram of experimental demodulator



(a) (b)

Fig. 3 - Response of system to pulse and bar test waveforms

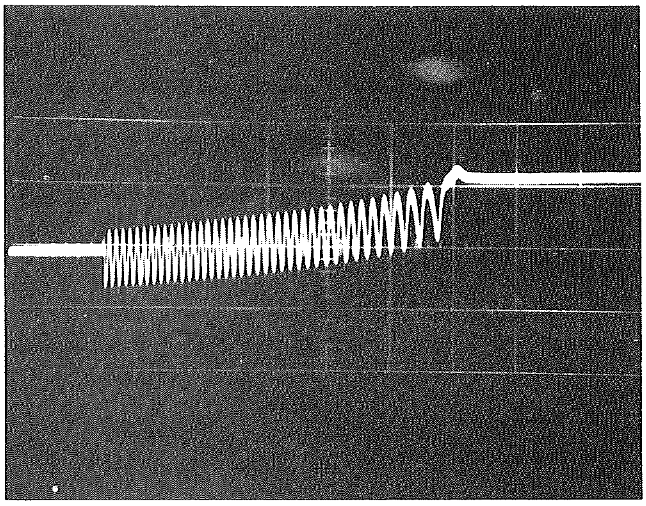
(a) response to 2T pulse and bar with 10T chrominance pulse channel 1 : (upper) channel 2 : (lower)
(b) response of system (channel 1) to 2T pulse showing K rating graticule: outer limits : 4% rating inner limits : 2% rating



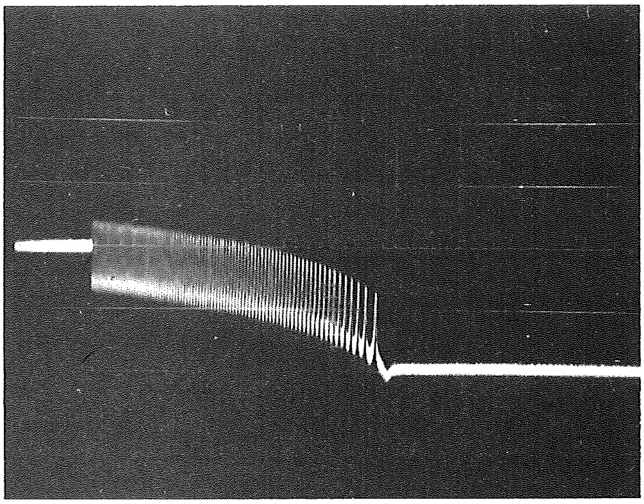
(a) (b) (c)

Fig. 4 - Transient response of the phase-lock loop. The response to a step in input signal phase is shown for different values of loop damping factor, σ,

Horizontal scale : 200 μs/div
Vertical scale = oscillator control voltage
(a) Underdamped, $\sigma \approx 0{\cdot}5$ (b) Critically damped, $\sigma \approx 1{\cdot}0$
(c) Overdamped, $\sigma \approx 2{\cdot}0$



(a)



(b)

Fig. 5 - Oscillator control voltage, showing pull-in waveform for initial frequency errors of −23 kHz and +50 kHz

(*a*) δf = −23 kHz

Horizontal scale = 0·5 ms/div Vertical scale = 0·5 volts/div

(*b*) δf = +50 kHz

Horizontal scale = 1 ms/div Vertical scale = 0·5 volts/div

The frequency response of the system was uniform to within 1·5 dB from 10 Hz to 5·5 MHz. The response of the system to a 625-line 2T pulse and bar test waveform is shown in Fig. 3. Fig. 3(*a*) shows an 8% loss of height of the 2T pulse with respect to the bar, and also a loss of about 1 dB at the colour subcarrier frequency in the 10T chrominance pulse. The 2T pulse is shown superimposed on a K-rating graticule in Fig. 3(*b*); the rating is shown to be less than 2%.

When random white noise was introduced between the modulator and demodulator, the picture quality impairment caused by crosstalk depended greatly on the damping of the phase-lock loop. Subjective tests indicated that a loop damping factor (see Appendix, Section 8.3.3) of approximately unity, that is, a critically-damped response, gave optimum performance of the loop for minimum impairment of the signals by noise. The effect of the damping control on the impulse response of the loop is shown in Fig. 4.

Temperature compensation of the phase-locked oscillator was used to reduce the phase drift to less than 0·3° under normal signal conditions. The pull-in characteristics of the oscillator were found to be similar to those predicted theoretically (see Appendix). Fig. 5 shows the oscillator control voltage waveforms for two initial frequency errors. The pull-in time for a frequency error of 50 kHz is approximately 4·5 ms.

5. Discussion

The main problem associated with the instrumentation of a practical quadrature modulation system is the necessity of providing a transmission pass-band with attenuation and phase characteristics symmetrical about the carrier frequency. As shown in Section 2, a small asymmetry of these characteristics can result in a relatively high level of crosstalk between the two channels. Crosstalk introduced by this means is likely to be most significant on terrestrial s.h.f. links where multipath propagation may effect the attenuation and phase characteristics. Because the amplitude and phase of multipath signals may change with atmospheric conditions it is not possible to employ fixed compensation for this form of signal distortion. For satellite communication, multipath propagation is found to be much less significant; the crosstalk in a quadrature modulated signal arising from propagation effects could therefore be expected to be small.

Phase distortion of the r.f. signal causes crosstalk which is in phase with the modulation but whose magnitude may depend on the modulating frequency. The method of phase locking of the reference carrier oscillator used in the experimental demodulator removes any in-phase crosstalk at 6 MHz but, unless the phase response of the link is perfectly antisymmetric, in phase crosstalk will be introduced at low modulation frequencies, as shown in Section 2. Non-symmetry of the attenuation characteristic of the link causes crosstalk in which all frequency components present in the original modulation are shifted in phase by 90 degrees. It may be possible to reduce this form of crosstalk by the use of an automatic equaliser, which could, for example, correct the attenuation at the edges of the pass-band. A control signal for this purpose could be derived from the demodulator by adding an additional phase-sensitive detector to the pilot-carrier system to detect the quadrature component of the pilot signal. This signal would be proportional to the difference in attenuation between the two pilot signal sidebands.

As discussed in Section 3.2.3, there is a 180° phase ambiguity in the locking of the reference carrier oscillator, so that the polarity of the demodulated output signal may be incorrect when the oscillator is locked to the input signal.

In a practical system additional circuitry would be required to detect the presence of an inverted output signal and reverse the phase of the reference carrier.

6. Conclusions

The experimental equipment has shown the feasibility of using quadrature modulation for transmission of two independent wide-band signals, and has illustrated some of the difficulties in instrumentation of the system. Crosstalk between the two channels is likely to make this method of modulation unsuitable for analogue signals except perhaps in certain cases such as satellite communication or short-distance terrestrial links where the attenuation and phase characteristics of the link can be accurately controlled.

Quadrature modulation would however appear to be most useful for transmission of pulse-code modulated signals. In this application, crosstalk between channels of the quadrature-modulated signal would, of course, result in a reduction of the noise margin of the system, but a useful system design may be possible for normal types of link or for satellite broadcasting.

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APPENDIX

The Phase-Locked Oscillator

8.1. Requirements of the phase-lock loop

The requirements listed below are proposed as a tentative specification for a quadrature demodulator working at a v.h.f. intermediate frequency. The system considered is intended to carry two channels with a bandwidth of 5·5 MHz, suitable for colour television. The carrier frequency to be used in the experimental equipment is 57 MHz.

8.1.1. Hold-in range

The input carrier frequency of the demodulator will in practice be subject to slow frequency changes due to frequency drifts of the transmitter, and receiver local oscillator. The demodulator phase-locked oscillator must be able to follow any such frequency changes without losing phase lock. The possible frequency drift in the i.f. is assumed to be ±100 kHz. This applies also to the pull-in range which is more conveniently discussed separately (see Section 8.1.6).

8.1.2. Phase accuracy

As the frequency of the input carrier changes from the centre frequency of the unlocked oscillator the frequency of the oscillator must be able to track the input signal frequency with a small phase error. In order to maintain crosstalk at −40 dB the required demodulator reference phase accuracy is 0·57 degrees. This phase error must not be exceeded for the assumed frequency drifts of ±100 kHz.

8.1.3. Transient behaviour

The input signal may be subject to rapid changes of phase due to noise present in the original carrier signal before modulation and noise on any carriers used in subsequent frequency conversion. The demodulator reference oscillator must be able to follow rapid phase changes with a minimum of error. The time taken for the oscillator to reduce a phase error to the required accuracy depends on the damping of the control loop as well as its bandwidth, discussed below.

8.1.4. Noise bandwidth

In order that the locked oscillator can follow rapid changes in input carrier phase with minimum error the control loop must have a wide bandwidth. Increasing the bandwidth of the phase control loop also increases its susceptibility to phase jitter caused by noise introduced in the transmission path. In order to determine the optimum loop bandwidth the width (and form) of the noise spectrum of the carrier must be known. It would then be necessary to use a control-loop bandwidth which is large compared with this in order to ensure that the transmitted phase noise is not the dominant factor. It is expected that in a practical u.h.f. or s.h.f. system, the optimum noise bandwidth will be determined by the noise introduced in the frequency conversion process which would have an effect similar to that of phase jitter on the transmitter. In the experimental system no frequency conversion is used. The noise bandwidth must therefore be chosen somewhat arbitrarily, and a value of 2 kHz is chosen.

8.1.5. Phase jitter caused by noise

The subjective effect on a video signal of phase errors of the oscillator due to noise, as discussed above, would be noise-modulated crosstalk between channels. A possible specification for noise immunity would be that the degradation of picture quality due to noise-modulated crosstalk should not exceed that due to the noise present on the video signal. This specification requires knowledge of the relative subjective impairment caused by video noise and and noise-modulated crosstalk. In the absence of such information, it is assumed that the r.m.s. value of noise-modulated crosstalk should not exceed the r.m.s. video noise.

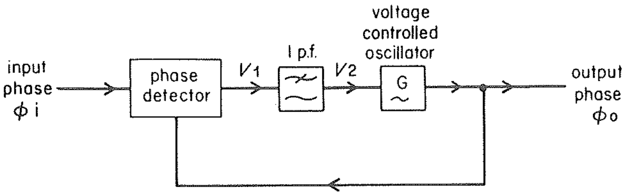

Fig. 6 - Simplified block diagram of the phase-lock loop

8.1.6. Pull-in range

When the demodulator is switched on there may be a frequency error between the input signal and the demodulator oscillator. In control systems, the hold-in range can often exceed the pull-in range by a considerable margin, but, for the present application, it is assumed that the oscillator must pull-in or hold-in for the same frequency error, namely, up to 100 kHz.

8.1.7. Pull-in time

The time taken for the loop to correct any frequency error up to the assumed maximum and bring the oscillator within the required phase tolerance should be less than 0·5 second.

8.2. Analysis of the phase-lock loop

A simplified phase-lock loop is shown in Fig. 6.

The oscillator will initially be assumed to be locked to the input reference signal.

In the following analysis frequent use will be made of the Laplace transform, which may be defined by the equation

$$G(s) = \int_0^\infty f(t) e^{-st}\, dt$$

where $G(s)$ is the Laplace transform of $f(t)$, and the variable s can be regarded as a complex frequency.

8.2.1. Phase detector

In the prototype equipment a simple diode phase detector was used. The output voltage from this phase detector is approximately $v_1(t) = K_d \cos[\phi_o(t) - \phi_i(t)]$ where ϕ_i and ϕ_o are the input and oscillator phases respectively, and K_d is a constant dependent on the input signal amplitude. At the designed operating signal level of the demodulator the value of K_d was 0·8 volts/radian. In the ideal locked condition of the loop ϕ_i and ϕ_o are in phase quadrature, but in practice there will be a small phase error. If ϕ_o' is defined as $\phi_o' = (\phi_o - 90^\circ)$, and $(\phi_o' - \phi_i)$ is assumed to be small, it is possible to make the approximation

$$v_1(t) = K_d[\phi_i(t) - \phi_o'(t)]$$

By substituting Laplace transforms in the above equation the transfer function of the phase detector may be written:

$$V_1(s) = K_d[\theta_i(s) - \theta_o(s)] \qquad (8.1)$$

where $V_1(s)$ is the Laplace transform of $v_1(t)$ and $\theta_i(s)$ and $\theta_o(s)$ are the Laplace transforms of $\phi_i(t)$ and $\phi_o'(t)$ respectively.

8.2.2. Low-pass filter

The low-pass filter is used to stabilise the loop and define the loop bandwidth.

If the Laplace transforms of the input and output voltages of the filter are $V_1(s)$ and $V_2(s)$ respectively the transfer function $F(s)$ of the filter may be defined:

$$V_2(s) = V_1(s).F(s) \qquad (8.1)$$

If it is necessary to provide d.c. amplification of the phase-detector output in order to achieve the required phase accuracy in the phase-lock loop, it is convenient to combine the d.c. amplifier and low-pass filter in the form of an active filter. In this case the d.c. gain of the filter F(o) may have a value exceeding unity.

8.2.3. Voltage controlled oscillator

In order to provide the oscillator deviation of ±100 kHz at a centre frequency of 57 MHz a varicap-diode-tuned LC oscillator was used in the prototype equipment. The frequency deviation δf of the oscillator from its centre frequency is assumed to be proportional to the control voltage over the frequency range considered, so that for a control voltage v_2:

$$2\pi\, \delta f = \frac{d\phi_o'(t)}{dt} = K_o v_2(t)$$

where K_o is the oscillator control constant.

By taking Laplace transforms this equation becomes:

$$s\theta_o(s) = K_o V_2(s) \qquad (8.3)$$

The value of the control constant, K_o, of the oscillator used in the prototype equipment was 10^6 radians/second/volt.

8.3. Design of the phase-lock loop

The design of the phase-lock loop will be outlined by considering in turn each of the requirements given in 8.1.

8.3.1. Hold-in range

If the phase-locked oscillator must remain in lock for frequency changes δf of up to ±100 kHz, the d.c. output voltage that must be available from the low-pass filter can be calculated:

$$v_2 = \pm 2\pi\, \delta f / K_o \text{ volts.}$$

If $K_o = 10^6$ radians/second/volt the filter must be capable of supplying an output voltage swing of ±0·63 volts.

8.3.2. Phase accuracy

If the deviation of the oscillator from its centre frequency is δf, the control voltage required is $2\pi\, \delta f/K_o$. If the phase error is not to exceed $\delta\theta$ degrees, the maximum output voltage of the phase detector v_1 is given by $v_1 = K_o\ \delta\theta.\pi/180$. The minimum d.c. gain, F(o), of the low-pass filter can therefore be calculated:

$$\text{since } V_2 = V_1 F(\text{o})$$

$$2\pi\, \delta f/K_o = K_o F(\text{o})\, \delta\theta.\pi/180.$$

If the phase error $\delta\theta$ must not exceed 0·57 degrees at a frequency deviation of 100 kHz, the minimum value of F(o) = 77. In order to allow for some offset error and for variation of the phase detector constant K_o with the signal level, a value of F(o) = 200 was chosen for the prototype design.

8.3.3. Transient behaviour of loop

From Sections 8.2.1, 8.2.2 and 8.2.3 we have:

$$V_1(s) = K_d[\theta_i(s) - \theta_o(s)] \qquad (8.1)$$

$$V_2(s) = V_1(s)\, F(s) \qquad (8.2)$$

$$s\theta_o(s) = K_o V_2(s) \qquad (8.3)$$

Combining (8.1), (8.2) and (8.3),

$$\theta_o(s) = \frac{K_o}{s} F(s)\, K_d[\theta_i(s) - \theta_o(s)]$$

Defining $H(s)$ as the closed loop transfer function = $(\theta_o(s))/(\theta_i(s))$.

$$H(s) = \frac{K_o K_d F(s)}{s + K_o K_d F(s)} \tag{8.4}$$

It will be shown later that, if a loop filter with the transfer function $F(s) = F(o)(1 + s\tau_1)/(1 + s\tau_2)$ is used the natural frequency and damping of the loop may be varied independently.

Substituting for $F(s)$ in (8.4)

$$H(s) = \frac{K(1 + s\tau_1)}{s(1 + s\tau_2) + K(1 + s\tau_1)}, \text{ where } K = K_d K_o F(o)$$

$$\text{therefore } H(s) = \frac{\frac{K}{\tau_2}(1 + s\tau_1)}{s^2 + \frac{s}{\tau_2}(1 + K\tau_1) + \frac{K}{\tau_2}} \tag{8.5}$$

The characteristic equation, which defines the transient behaviour of the loop, is

$$s^2 + \frac{s(1 + K\tau_1)}{\tau_2} + \frac{K}{\tau_2} = 0,$$

or, in terms of the loop natural frequency, ω_n, and loop damping factor, σ,

$$s^2 + 2\sigma\omega_n s + \omega_n^2 = 0$$

where $\omega_n = (K/\tau_2)^{1/2}$ and

$$\sigma = \frac{1 + K\tau_1}{2\tau_2}(\tau_2/K)^{1/2}$$

$$\simeq \tfrac{1}{2}\tau_1 (K/\tau_2)^{1/2} \text{ for } K\tau_1 \gg 1.$$

Substituting for σ and ω_n in Equation (8.5).

$$H(s) \simeq \frac{\omega_n^2 + 2\sigma\omega_n s}{\omega_n^2 + 2\sigma\omega_n s + s^2} \tag{8.6}$$

The time taken for the loop to stabilise after a transient input depends greatly on the value of σ. If σ is less than unity the transient response shows overshoots before stabilising. For σ greater than unity the response is overdamped and the decay of the transient response is exponential. The impulse response of the loop with different damping factors is shown in Fig. 4. A damping factor of about 0·7 is usually found to be optimum for a rapid settling time of the loop.

8.3.4. Noise bandwidth

The noise bandwidth B_N of a loop with transfer function $H(s)$ is defined as the bandwidth of the ideal sharp-cut low-pass filter which produces the same amount of noise power at its output as does the linear system whose transfer function is $H(s)$, when the input to both systems is noise with a flat spectral density. Applying this definition to Equation (8.6), Viterbi[5] has shown that

$$B_N = \frac{\omega_n(1 + 4\sigma^2)}{8\sigma}$$

For B_N = 2 kHz and σ = 0·7 we deduce $\omega_n = 2{\cdot}5 \times 10^4$ rad/s.

8.3.5. Phase jitter due to noise

The phase modulation of the locked oscillator by noise present at the input may be analysed by assuming the noise component is small compared with the signal; the noise may be expressed as $N(t) = a(t)\cos\omega_p t + b(t)\sin\omega_p t$, where ω_p is the frequency of the pilot signal. $a(t)$ and $b(t)$ are independent time-varying parameters and $\overline{a^2} = \overline{b^2}$.

$$\text{The total noise power } \overline{N^2} = \overline{(a\cos\omega_p t + b\sin\omega_p t)^2}$$

$$= \tfrac{1}{2}\overline{a^2} + \tfrac{1}{2}\overline{b^2}$$

$$\text{hence } \overline{N^2} = \overline{a^2} = \overline{b^2}$$

If the pilot signal is of the form $p\cos\omega_p t$, the signal plus noise = $[a(t) + p]\cos\omega_p t + b(t)\sin\omega_p t$, and the instantaneous input phase error due to noise is given by

$$\phi \approx \tan\phi = \frac{b}{p + a}$$

$$\approx \frac{b}{p} \text{ for } p \gg a$$

Therefore if the r.m.s. noise voltage present in the pilot signal is N, the r.m.s. phase error ϕ is approximately N/p.

If we define:

N_V = r.m.s. noise voltage in video bandwidth

B_N = Loop noise bandwidth

B_V = Video bandwidth

h = Ratio of r.m.s. pilot carrier amplitude to peak-to-peak video amplitude.

V_p = Peak-to-peak video amplitude

$$\text{r.m.s. noise in loop bandwidth} = N = N_V\sqrt{\frac{B_N}{B_V}}$$

$$p = \sqrt{2}hV_p$$

$$\text{Therefore } \phi \text{ r.m.s.} = \frac{N_V}{\sqrt{2}hV_p}\sqrt{\frac{B_N}{B_V}}\ \frac{360}{2\pi}\text{degrees}$$

The peak-to-peak amplitude of crosstalk caused by a small phase error ϕ radians = ϕV_p. Therefore, to comply with the specification for phase jitter (Section 8.1.5),

$$\phi_{\mathrm{r.m.s.}} V_{\mathrm{p}} \leqslant N_{\mathrm{V}}$$

Hence $N_{\mathrm{V}} \geqslant \left(\frac{N_{\mathrm{V}}}{\sqrt{2}h}\right)\left(\sqrt{\frac{B_{\mathrm{N}}}{B_{\mathrm{V}}}}\right)$ or $B_{\mathrm{N}} \leqslant 2B_{\mathrm{V}} h^2$

where $B_{\mathrm{N}} = 2 \times 10^3$ Hz.

$B_{\mathrm{V}} = 6 \times 10^6$ Hz.

Thus $h \geqslant 0{\cdot}0128$ so that the minimum level of pilot signal with respect to peak video is −38 dB, at which level the r.m.s. crosstalk equals the r.m.s. video noise.

The demodulator design is simplified if the pilot carrier level is raised to −20 dB, and the requirements of the pilot frequency band-pass filter are then much less stringent. The increased level of pilot signal would cause only a slight reduction in transmitter power efficiency, and would enable the crosstalk to be reduced to a level small compared with the noise in the video channels.

8.3.6. Pull-in range

When the loop is in the unlocked condition the linear form of the transfer function is not valid, and a formula taking into account the sine law of the phase detector must be used.

Several methods have been devised to obtain approximate expressions for the pull-in range of second order phase locked loops.[4,5,6] One formula in a particularly useful form is used by Gardner,[8] derived from a method due to Richman:[4]

Pull-in range $\Delta f_p = (1/\pi)\,(\sigma\omega_{\mathrm{n}} K - \tfrac{1}{2}\omega_{\mathrm{n}}^{\,2})^{\frac{1}{2}}$
With a natural frequency ω_{n} $2{\cdot}5 \times 10^4$ rad/s,

a damping factor $\sigma = 0{\cdot}7$ and $F(\mathrm{o}) = 200$:

$$K = K_{\mathrm{d}} K_{\mathrm{o}} F(\mathrm{o}) = 1{\cdot}6 \times 10^8$$

$$\Delta f_p \approx 0{\cdot}53 \times 10^6 \text{ Hz.}$$

The methods used for calculating the pull-in range of the oscillator assume a loop with a single filter with a transfer function of the type described. The practical system, using a pilot carrier locking system, requires additional pilot-frequency band-pass filters in the loop to provide rejection of the video and noise signals.

Leonhardt and Fleischmann[7] have published work on loops containing additional filters. Using Leonhardt and Fleischmann's analysis it should be possible to provide the required pull-in range if the bandwidth of the pilot-carrier filters exceeds 110 kHz.

8.3.7. Pull-in time

The time taken for the loop to pull into lock from an initial frequency error δf is calculated by Viterbi[6] as:

$$t_p \approx \frac{(2\pi\delta f)^2}{2\sigma\omega_{\mathrm{n}}^{\,3}}\,.$$

This is not accurate that the limits of the pull-in range, where δf approaches Δf_p. The time may be calculated for $\delta f = \tfrac{1}{2}\Delta f_p = 50 \times 10^3$ Hz.

If $\sigma = 0{\cdot}7$ and $\omega_{\mathrm{n}} = 2{\cdot}5 \times 10^4$ rad/s

$$t_p = \frac{(50 \times 2\pi \times 10^3)^2}{1{\cdot}4(2{\cdot}5 \times 10^4)^3}$$

$$= 4{\cdot}5 \times 10^{-3} \text{ seconds.}$$

This is well within the specification of the system.

8.4. A practical loop filter

Taking the values derived in Section 8.3:

$\omega_{\mathrm{n}} = 2{\cdot}5 \times 10^4$ rad/s	(Section 8.3.4)
$\sigma = 0{\cdot}7$	(Section 8.3.3)
$F(\mathrm{o}) = 200$	(Section 8.3.2)

and using the parameter equations from Section 6.3.3, viz.

$$\omega_{\mathrm{n}} = \sqrt{\frac{K}{\tau_2}}$$

and $$\sigma = \frac{\tau_1}{2}\sqrt{\frac{K}{\tau_2}}$$

where $K = K_{\mathrm{o}} K_{\mathrm{d}} F(\mathrm{o}) = 1{\cdot}6 \times 10^8$,

we obtain $\tau_2 = 0{\cdot}25$ s.

and $\tau_1 = 5{\cdot}5 \times 10^{-5}$ s.

The required filter transfer function is

$$F(s) = F(\mathrm{o})\frac{1 + \tau_1 s}{1 + \tau_2 s}$$

$$F(s) = 200\,\frac{1 + 5{\cdot}5 \times 10^{-5} s}{1 + 0{\cdot}25 s}$$

A suitable transfer function may be realised with the circuit shown in Figure 7 and was used for the experimental model.

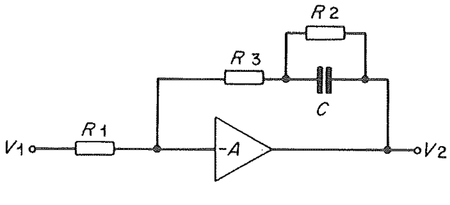

Fig. 7 - Practical loop filter

$R_3 C = 5{\cdot}5 \times 10^{-5}$ s $\quad R_2 C = 0{\cdot}25$ s

$R_2/R_1 = 200 \quad A \geqslant 200$

The transfer function of this circuit is approximately

$$F(s) = -200\frac{1 + 5{\cdot}5 \times 10^{-5}s}{1 + 0{\cdot}25s}$$

Owing to the phase reversal in this loop filter the polarity of K_d or K_o must be made negative in order to satisfy the design equations given in Section 8.3.

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